

Music Derived from Other Sources

Clarence Barlow

Corwin Professor and Head of Composition, Music Department, University of California, Santa Barbara

Abstract

One imagines composed music as originating in the mind of an author. Indeed, I have for the last five decades been composing music in this fashion. However, for the last four decades I have also been repeatedly attracted to various methods of deriving music from sources both inside and outside the field. These sources have been linguistic, acoustic (field recordings), visual and mathematical (parametric formulae) as well as other works of music. For most of these operations I have resorted to strict algorithmic means and the use of computer programming. In this paper I will outline the sources mentioned above and go a little into detail in some of them. For instance the linguistic: I have used text orthography, spectral analyses of human speech, digital recordings of the human voice, and synthetic semantic structures. In the case of the visual, it was the conversion of pixellated photographs, city silhouettes, abstract films and geometric models as well as the visualization of algorithmically generated music that played a central role in the compositional plan. In other words, the paper will provide a general overview and examples. This is the first time I have covered more than one type of source in a single article.

Keywords: linguistics, orthography, semantics, synthstrumentation, digital sound processing, field recordings, image sonification, harmonicity, metricity.

Linguistic Sources

The two main sources I have resorted to in this field are *orthography* and *speech recordings*.

For the former, it was the spelling of a text chosen at will which provided the material for composition. My prime example of this usage is the composition *Textmusic* (1971). In its most basic form, this work is a list instructing a composer (who could be someone other than myself) to allocate the letters of a text, each of them once only, to a piano keyboard, starting in the middle and moving outwards in a zigzag way. This procedure is to be executed three times, once on the black keys, once on the white, and once disregarding the key colour. Thereafter the text can be "played", to wit, in single letters, or in whole syllables at a time as chords, or similarly in words, phrases and sentences. One can freely move from one text level as just described to another or from one key colour system to another between any two successive words. Further aspects such as loudness, length and right pedal depression are also left to the composer's discretion, to be effected, when desired, at syllable change.

As an example of this method I cite *Textmusic #6* (1973), based on the text *Ping* by Samuel Beckett, the opening line of which is "All known all white bare white body fixed one yard legs joined like sewn". Accordingly the nineteen letters ALKNOWHITEBRDYFXGSJ contained in this sentence are allocated in turn to as many black (pentatonic), white (diatonic) and mixed-colour keys (chromatic). Further letters like QUV... are allocated as they appear in the text. Fig.1 shows the allocation of the first nineteen letters in chromatic mode at top left and right as well as the opening notes upto "joined".

See also reference [2].

Fig.1 – *Textmusic #6*: Letter allocation and beginning of piece.

The figure shows a piano keyboard with letters F, B, H, A, N, E, Y, S on the white keys and J, G, D, T, O, K, L, W, I, R, X on the black keys. To the right, a musical staff in G major (one sharp) contains the melody for the phrase "All known all white bare white body fixed one yard legs joined..." with the letters A, L, K, N, O, W, H, I, T, E, B, R, D, Y, F, X, G, S, J written below the notes. Below this, an excerpt from the piece is shown in two systems. The first system contains the notes for "A - L - L K - N - O - W - N A - L - L WHITE B - A - R - E W - H - I - T - E B O D Y _". The second system contains the notes for "F - I - X - E - D O - N - E Y A R D L - E - G - S J - O - I - N - E - D". The word "Red." is written above the first and third measures of the second system, and an asterisk is above the second and fourth measures.

As examples of the use of speech recordings for composition, I will cite two of several techniques I developed for the purpose, *synthrummentation* ("sound synthesis through instrumentation", 1981) and *semantico-grammatical linguistic score synthesis* (2003).

The former converts the spectral analysis of speech into a music score which, when played, brings forth a sound similar to the source, to the point of some words being actually recognisable. As one of several examples of my use of this technique, the three phrases "why me?", "no money" and "my way" are scored for piano and strings in my large orchestral piece *Orchideae Ordinariae* (1989) as shown in Fig.2. The middle phrase "no money" is particularly recognisable in performance.

See also references [1] and [8].

Fig.2 - *Orchideae Ordinariae*: three spoken phrases scored for instruments.

The figure shows a page of a musical score for three phrases: "why me?", "no money", and "my way". The score is for a large orchestra, including Piano (Pfte), Violins (VI.1a+b, VI.1c+d, VI.1e+f, VI.2a+b, VI.2c+d, VI.2e+f), Violas (Va.a-c, Va.d+e), Cellos (Vc.a+b, Vc.c+d), and Double Basses (Cb.a+b, Cb.c+d, Cb.e+f). The score is in 4/4 time and starts at measure 66. The three phrases are clearly marked at the top of the score.

As an example of semantico-grammatical linguistic score synthesis I cite *Progéthal Percussionian for the Advanced Beginner* (2003), a piece written in a "language" crafted for percussion instruments, termed "Progéthal Percussionian": here sentences of a given text are grammatically parsed into parts of speech – nouns, verbs etc. with their several attributes – and then converted into a score by a fixed set of rules, using a vocabulary developed with the help of Roget's Thesaurus (hence the name of the language). The piece purports to be a course in the language. Fig.3 shows the translation of the first sentence of Hamlet's Soliloquy – the low note (played on the bass drum) near the beginning of bar 3 for instance means "not": compare this bar to bar 1 ("to be").

Fig.3 - *Progéthal Percussionian for the Advanced Beginner*: Hamlet's Soliloquy (opening).

I. 2BR02B-2
For two players

parlando

Metal+Air+Wood...
Plastic+Glass+Skin...

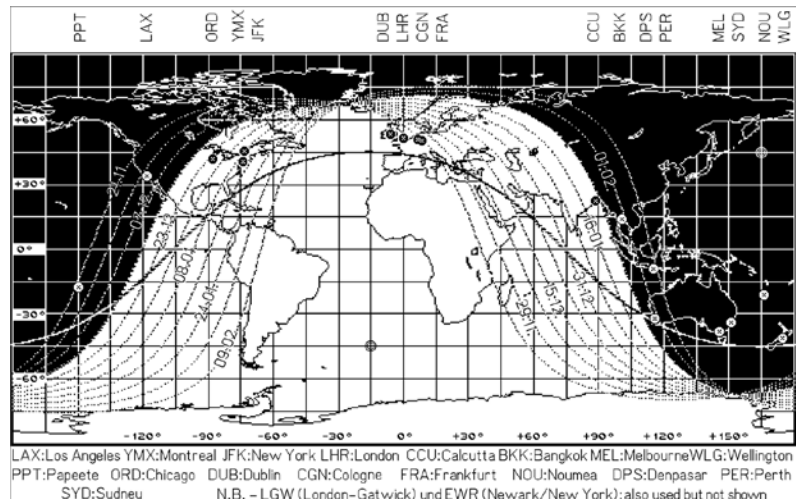
To be, or not to be, that is the question:

Acoustic Sources

Here I refer to *field recordings* made with suitable technical equipment in various places. My octaphonic electronic work *CCU* (1980) layers seven of twenty hours of stereo recordings made during three months in the winter of 1977-78 at sixty venues in Calcutta, both in- and outdoors, to form a 48-minute piece reflecting the course of a hypothetical 24-hour day in that city. The loudspeaker panning matches the location of the venues, north in front, south behind etc.; the distance of the venue from the city centre determines the separation of the two speakers diffusing the stereo recording, the further the venue from the centre, the closer together the speakers.

On the other hand, the octaphonic 41-minute *Zero Crossing* (2001) comprises 83 recordings made daily at the same stellar time in 31 places on a three-month trip around the world in the winter of 1999-2000. Each of the recordings was carefully whittled down to an "edition" lasting exactly two minutes, the central 30 seconds of which (the "core") is played at the original volume with the preceding and following 45 seconds constituting a fade-in and -out. Fig.4 shows a map of the world with my airports of call, all situated within 30 degrees north of a great circle poled in the North-West Pacific and the South Atlantic.

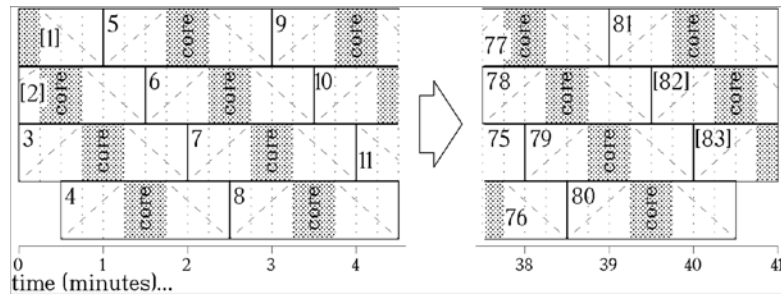
Fig.4 – *Zero Crossing*: equirectangular World Map showing airports of call and neighbouring great circle.



The system of the layering of the 83 editions is shown in Fig.5; each edition is faded in upto the core and faded out thereafter. Incomplete editions are numbered in square brackets.

See also reference [3].

Fig.5 – *Zero Crossing*: the system of edition layering.

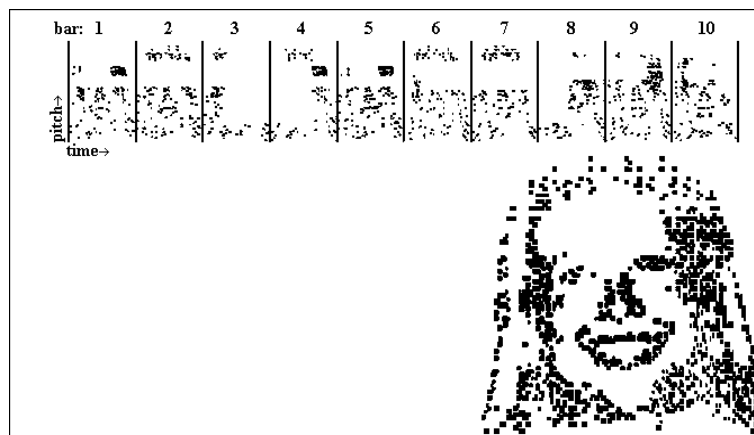


Visual Sources

To illustrate the derivation of my music from the visual I have chosen the *Chaconne*, the main part of my piano piece *Kuri Suti Bekar* (1998), written for the pianist Kristi Becker, whose name, pronounced in Japanese, inspired the title. Her portrait photograph, scanned into an 88x88-pixel matrix, was mapped ten times to form ten pages of a music score, whereby the 88 horizontal pixels match an equal number of pulses in time, and the 88 vertical pixels correspond to the 88 piano keys. For each mapping, pixels were selected according to a set of pre-fixed constraints, the three most significant being 1) their being transected by two sets of 120 radial lines three degrees apart, emanating from two arbitrary points near the top left and right of the image and rotating from one mapping to the next (20 of these lines were ignored – see below), 2) a harmonic derivation from my earlier piano piece *...until... #5* (1972/74) in $11/16$ metre, often played by Ms Becker (each mapping scans a set of eight bars or 88 pulses of this piece at a time), and 3) a phonetic spectral analysis, serving as a sonic filter, of the three Bengali words coincidentally forming the title: "kuri" (twenty), "suti" (cotton fibers) and "bekar" (unemployed) – referring to the 20 unused lines. Fig.6 shows at the top pitch-time images of the ten mappings or bars, which when superimposed yield the graph at bottom right, clearly resembling the pixellated photograph.

See also reference [7].

Fig.6 – The ten mappings forming the *Chaconne* of *Kuri Suti Bekar* shown singly (above) and superimposed (bottom right).

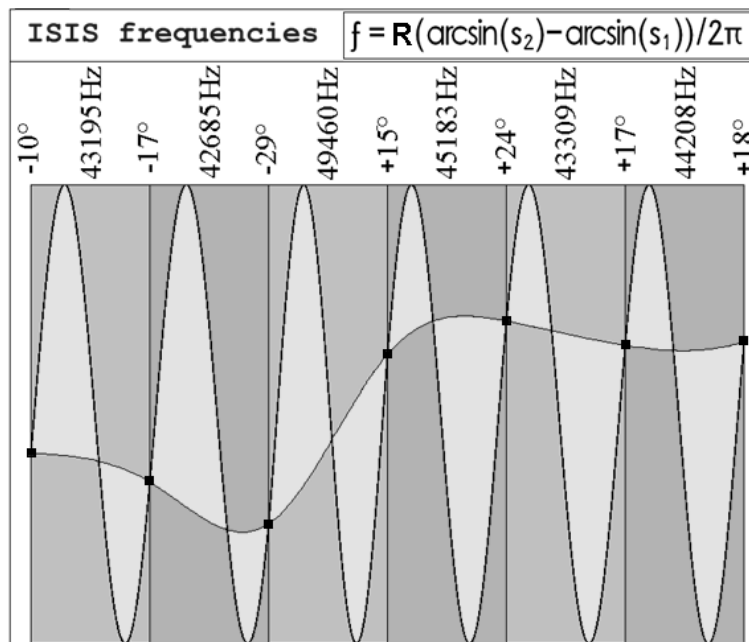


Mathematical Sources

Two sources are demonstrated here, one – *ISIS* – in the realm of Digital Sound Processing and the other – *the quantification of harmony and metre* – in the realms of pitch and pulse, melody and rhythm.

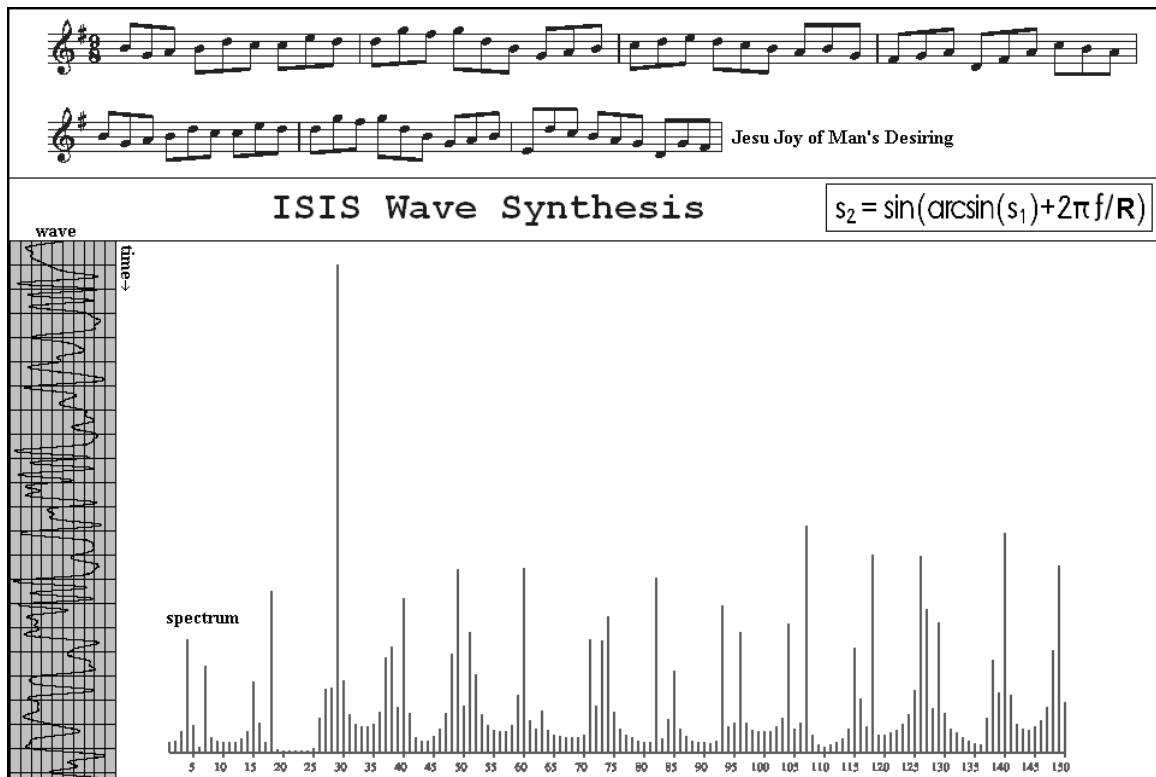
ISIS – for "Intra-Samplar Interpolation of Sinusoids" – resulted from my having doubted, already forty years ago, that white noise is the sum of all frequencies. I argued that if this were the case, each frequency, one of an infinite number, would have to be of zero amplitude, in order not to add up in loudness to an infinite value. The model I preferred for white noise was that of a single sine tone moving randomly to all possible frequencies at enormously high speed. After some experiments with MIDI in the 1980s that seemed to bear out this supposition, I finally got down in 2001 to the mathematics of determining the variable sine-tone frequency of a sound wave in general. Presuming that it was sufficient to set the speed of frequency change at the sampling rate (currently commonly 44.1 or 48 KHz), I interpolated hypothetical sine-wave segments between the samples of a sound wave, such that each segment would pass through the limiting values ± 1 once each and the final phase of one sine segment and the initial phase of the next would be equal. Fig.7 shows a set of seven samples (square dots) connected by a spline, but more significantly by six sine segments, the frequencies and inter-segment common phase of which are given at the top. The formula yielding these frequencies is seen at top right, where " f " is the frequency, " R " the sampling rate and " s_1 " and " s_2 " the values (within ± 1) of two successive samples. It is easy to see why the frequencies are centred around the sampling rate: if two successive samples are equal in value, the sine segment between them will form exactly one period of the sampling rate frequency.

Fig.7 – Formula for *ISIS*-analysis of a sound wave with a graphic example.



This extraction of sine frequencies from a sound wave is tantamount to converting the latter into a "melody", albeit ultrafast and ultrasonic. However, it is possible to transpose this melody down into the audible range and to slow it down to a perceptible speed. It is equally possible to transpose any given melody in pitch and speed up to the sampling rate, thus converting it into a sound wave, as in Fig.8, which shows the process applied to J. S. Bach's *Jesu Joy of Man's Desiring* by the reverse formula for *ISIS*-synthesis. The resulting sound wave and its spectrum are also shown.

Fig.8 – Bach's *Jesu Joy of Man's Desiring* converted to a sound wave (bottom left) with spectrum



ISIS thus combines the domains of melody and rhythm with that of timbre to form a continuum.

See also reference [6].

About the quantification of harmony and metre:

In the late 1970s, in search of an effective way of estimating how harmonic a pitch interval is (including so-called micro-intervals), I studied observations by Pythagoras, Hindemith, Partch and other experts as well as some relevant work in the field by Euler. None of the above helped me in making an exact and usable measurement, leading me to delve into research of my own. The result: a formula for *harmonicity*, based on a coefficient of *indigestibility* of natural numbers, a coefficient based on both their size and divisibility. Both formulae and corresponding tables are shown in Fig.9. Take for instance the numbers 7, 8 and 9: the indigestibility of 7 (10.3), a prime, is high in comparison to that of 8 (3.0), a number divisible down to the smallest prime 2. The indigestibility of 9 (5.3), the square of 3, lies between the two previously named values. Moving on, the formula for harmonicity is defined as the reciprocal of the sum of the indigestibilities of the numerator and denominator of the ratio or fraction forming the interval, with one added feature: if the numerator is more digestible, i.e. less indigestible than the denominator (as in 4/3), the upper note becomes the root of the interval and the harmonicity is allotted a negative value, as determined by the plus-minus function " $\text{sgn}(\xi(Q) - \xi(P))$ ", where "P" is the denominator, "Q" the numerator and " ξ " the indigestibility (the harmonicity of 4/3 or 3:4 is -0.214, as seen in the two right-most columns of Fig.9).

Fig.9 – Formulae and tables for Indigestibility (upper left) and Harmonicity (lower left).

| $\xi(N) = 2 \sum_{r=1}^{\infty} \left\{ \frac{n_r (p_r - 1)^2}{p_r} \right\}$ <p>whereby:</p> <ol style="list-style-type: none"> $N = \prod_{r=1}^{\infty} p_r^{n_r}$ $N, n_r, p \in$ natural numbers $p \in$ prime numbers | | <table border="1"> <thead> <tr> <th>Indigestibility $\xi(N)$ of the natural numbers 1-16</th> <th colspan="10">Complete Intraoctave Intervals upwards of Harmonicity 0.05</th> </tr> <tr> <th>N</th> <th>$\xi(N)$</th> <th>Interval-size (Ct)</th> <th>2</th> <th>3</th> <th>5</th> <th>7</th> <th>11</th> <th>13</th> <th>Number-ratio</th> <th>Harmonicity</th> </tr> </thead> <tbody> <tr><td>1</td><td>0.000000</td><td>0.000</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1:1</td><td>∞</td></tr> <tr><td>2</td><td>1.000000</td><td>70.672</td><td>-3</td><td>-1</td><td>+2</td><td>0</td><td>0</td><td>0</td><td>24:25</td><td>+0.054152</td></tr> <tr><td>3</td><td>2.666667</td><td>111.731</td><td>+4</td><td>-1</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>15:16</td><td>-0.076531</td></tr> <tr><td>4</td><td>2.000000</td><td>182.404</td><td>+1</td><td>-2</td><td>+1</td><td>0</td><td>0</td><td>0</td><td>9:10</td><td>+0.078534</td></tr> <tr><td>5</td><td>6.400000</td><td>203.910</td><td>-3</td><td>+2</td><td>0</td><td>0</td><td>0</td><td>0</td><td>8:9</td><td>-0.120000</td></tr> <tr><td>6</td><td>3.666667</td><td>231.174</td><td>+3</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>7:8</td><td>-0.075269</td></tr> <tr><td>7</td><td>10.285714</td><td>266.871</td><td>-1</td><td>-1</td><td>0</td><td>+1</td><td>0</td><td>0</td><td>6:7</td><td>+0.071672</td></tr> <tr><td>8</td><td>3.000000</td><td>294.135</td><td>+5</td><td>-3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>27:32</td><td>-0.076923</td></tr> <tr><td>9</td><td>5.333333</td><td>315.641</td><td>+1</td><td>+1</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>5:6</td><td>-0.099338</td></tr> <tr><td>10</td><td>7.400000</td><td>386.314</td><td>-2</td><td>0</td><td>+1</td><td>0</td><td>0</td><td>0</td><td>4:5</td><td>+0.119046</td></tr> <tr><td>11</td><td>18.181818</td><td>407.820</td><td>-6</td><td>+4</td><td>0</td><td>0</td><td>0</td><td>0</td><td>64:81</td><td>+0.060000</td></tr> <tr><td>12</td><td>4.666667</td><td>427.373</td><td>+5</td><td>0</td><td>-2</td><td>0</td><td>0</td><td>0</td><td>25:32</td><td>-0.056180</td></tr> <tr><td>13</td><td>22.153846</td><td>435.084</td><td>0</td><td>+2</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>7:9</td><td>-0.044024</td></tr> <tr><td>14</td><td>11.285714</td><td>470.781</td><td>-4</td><td>+1</td><td>0</td><td>+1</td><td>0</td><td>0</td><td>16:21</td><td>+0.058989</td></tr> <tr><td>15</td><td>9.066667</td><td>498.045</td><td>+2</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>3:4</td><td>-0.214286</td></tr> <tr><td>16</td><td>4.000000</td><td>519.551</td><td>-2</td><td>+3</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>20:27</td><td>-0.060976</td></tr> <tr><td></td><td></td><td>568.717</td><td>-1</td><td>-2</td><td>+2</td><td>0</td><td>0</td><td>0</td><td>18:25</td><td>+0.052265</td></tr> <tr><td></td><td></td><td>582.512</td><td>0</td><td>0</td><td>-1</td><td>+1</td><td>0</td><td>0</td><td>5:7</td><td>-0.059922</td></tr> <tr><td></td><td></td><td>590.224</td><td>-5</td><td>+2</td><td>+1</td><td>0</td><td>0</td><td>0</td><td>32:45</td><td>+0.059761</td></tr> <tr><td></td><td></td><td>609.776</td><td>+6</td><td>-2</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>45:64</td><td>-0.056391</td></tr> <tr><td></td><td></td><td>617.488</td><td>+1</td><td>0</td><td>+1</td><td>-1</td><td>0</td><td>0</td><td>7:10</td><td>-0.056543</td></tr> <tr><td></td><td></td><td>680.449</td><td>+3</td><td>-3</td><td>+1</td><td>0</td><td>0</td><td>0</td><td>27:40</td><td>+0.057421</td></tr> <tr><td></td><td></td><td>701.955</td><td>-1</td><td>+1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2:3</td><td>+0.272727</td></tr> <tr><td></td><td></td><td>729.219</td><td>+5</td><td>-1</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>21:32</td><td>-0.055703</td></tr> <tr><td></td><td></td><td>764.916</td><td>+1</td><td>-2</td><td>0</td><td>+1</td><td>0</td><td>0</td><td>9:14</td><td>+0.060172</td></tr> <tr><td></td><td></td><td>772.627</td><td>-4</td><td>0</td><td>+2</td><td>0</td><td>0</td><td>0</td><td>16:25</td><td>+0.059524</td></tr> <tr><td></td><td></td><td>792.180</td><td>+7</td><td>-4</td><td>0</td><td>0</td><td>0</td><td>0</td><td>81:128</td><td>-0.056504</td></tr> <tr><td></td><td></td><td>813.686</td><td>+3</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>5:8</td><td>-0.106385</td></tr> <tr><td></td><td></td><td>884.359</td><td>0</td><td>-1</td><td>+1</td><td>0</td><td>0</td><td>0</td><td>3:5</td><td>+0.110294</td></tr> <tr><td></td><td></td><td>905.865</td><td>-4</td><td>+3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>16:27</td><td>+0.083333</td></tr> <tr><td></td><td></td><td>933.129</td><td>+2</td><td>+1</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>7:12</td><td>-0.066879</td></tr> <tr><td></td><td></td><td>968.826</td><td>-2</td><td>0</td><td>0</td><td>+1</td><td>0</td><td>0</td><td>4:7</td><td>+0.081395</td></tr> <tr><td></td><td></td><td>996.090</td><td>+4</td><td>-2</td><td>0</td><td>0</td><td>0</td><td>0</td><td>9:16</td><td>-0.107143</td></tr> <tr><td></td><td></td><td>1017.596</td><td>0</td><td>+2</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>5:9</td><td>-0.085227</td></tr> <tr><td></td><td></td><td>1086.269</td><td>-3</td><td>+1</td><td>+1</td><td>0</td><td>0</td><td>0</td><td>8:15</td><td>-0.082375</td></tr> <tr><td></td><td></td><td>1125.338</td><td>+4</td><td>+1</td><td>-2</td><td>0</td><td>0</td><td>0</td><td>25:48</td><td>-0.051370</td></tr> <tr><td></td><td></td><td>1137.039</td><td>+1</td><td>+3</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>14:27</td><td>-0.051852</td></tr> <tr><td></td><td></td><td>1200.000</td><td>+1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1:2</td><td>+1.000000</td></tr> </tbody> </table> | Indigestibility $\xi(N)$ of the natural numbers 1-16 | Complete Intraoctave Intervals upwards of Harmonicity 0.05 | | | | | | | | | | N | $\xi(N)$ | Interval-size (Ct) | 2 | 3 | 5 | 7 | 11 | 13 | Number-ratio | Harmonicity | 1 | 0.000000 | 0.000 | 0 | 0 | 0 | 0 | 0 | 0 | 1:1 | ∞ | 2 | 1.000000 | 70.672 | -3 | -1 | +2 | 0 | 0 | 0 | 24:25 | +0.054152 | 3 | 2.666667 | 111.731 | +4 | -1 | -1 | 0 | 0 | 0 | 15:16 | -0.076531 | 4 | 2.000000 | 182.404 | +1 | -2 | +1 | 0 | 0 | 0 | 9:10 | +0.078534 | 5 | 6.400000 | 203.910 | -3 | +2 | 0 | 0 | 0 | 0 | 8:9 | -0.120000 | 6 | 3.666667 | 231.174 | +3 | 0 | 0 | -1 | 0 | 0 | 7:8 | -0.075269 | 7 | 10.285714 | 266.871 | -1 | -1 | 0 | +1 | 0 | 0 | 6:7 | +0.071672 | 8 | 3.000000 | 294.135 | +5 | -3 | 0 | 0 | 0 | 0 | 27:32 | -0.076923 | 9 | 5.333333 | 315.641 | +1 | +1 | -1 | 0 | 0 | 0 | 5:6 | -0.099338 | 10 | 7.400000 | 386.314 | -2 | 0 | +1 | 0 | 0 | 0 | 4:5 | +0.119046 | 11 | 18.181818 | 407.820 | -6 | +4 | 0 | 0 | 0 | 0 | 64:81 | +0.060000 | 12 | 4.666667 | 427.373 | +5 | 0 | -2 | 0 | 0 | 0 | 25:32 | -0.056180 | 13 | 22.153846 | 435.084 | 0 | +2 | 0 | -1 | 0 | 0 | 7:9 | -0.044024 | 14 | 11.285714 | 470.781 | -4 | +1 | 0 | +1 | 0 | 0 | 16:21 | +0.058989 | 15 | 9.066667 | 498.045 | +2 | -1 | 0 | 0 | 0 | 0 | 3:4 | -0.214286 | 16 | 4.000000 | 519.551 | -2 | +3 | -1 | 0 | 0 | 0 | 20:27 | -0.060976 | | | 568.717 | -1 | -2 | +2 | 0 | 0 | 0 | 18:25 | +0.052265 | | | 582.512 | 0 | 0 | -1 | +1 | 0 | 0 | 5:7 | -0.059922 | | | 590.224 | -5 | +2 | +1 | 0 | 0 | 0 | 32:45 | +0.059761 | | | 609.776 | +6 | -2 | -1 | 0 | 0 | 0 | 45:64 | -0.056391 | | | 617.488 | +1 | 0 | +1 | -1 | 0 | 0 | 7:10 | -0.056543 | | | 680.449 | +3 | -3 | +1 | 0 | 0 | 0 | 27:40 | +0.057421 | | | 701.955 | -1 | +1 | 0 | 0 | 0 | 0 | 2:3 | +0.272727 | | | 729.219 | +5 | -1 | 0 | -1 | 0 | 0 | 21:32 | -0.055703 | | | 764.916 | +1 | -2 | 0 | +1 | 0 | 0 | 9:14 | +0.060172 | | | 772.627 | -4 | 0 | +2 | 0 | 0 | 0 | 16:25 | +0.059524 | | | 792.180 | +7 | -4 | 0 | 0 | 0 | 0 | 81:128 | -0.056504 | | | 813.686 | +3 | 0 | -1 | 0 | 0 | 0 | 5:8 | -0.106385 | | | 884.359 | 0 | -1 | +1 | 0 | 0 | 0 | 3:5 | +0.110294 | | | 905.865 | -4 | +3 | 0 | 0 | 0 | 0 | 16:27 | +0.083333 | | | 933.129 | +2 | +1 | 0 | -1 | 0 | 0 | 7:12 | -0.066879 | | | 968.826 | -2 | 0 | 0 | +1 | 0 | 0 | 4:7 | +0.081395 | | | 996.090 | +4 | -2 | 0 | 0 | 0 | 0 | 9:16 | -0.107143 | | | 1017.596 | 0 | +2 | -1 | 0 | 0 | 0 | 5:9 | -0.085227 | | | 1086.269 | -3 | +1 | +1 | 0 | 0 | 0 | 8:15 | -0.082375 | | | 1125.338 | +4 | +1 | -2 | 0 | 0 | 0 | 25:48 | -0.051370 | | | 1137.039 | +1 | +3 | 0 | -1 | 0 | 0 | 14:27 | -0.051852 | | | 1200.000 | +1 | 0 | 0 | 0 | 0 | 0 | 1:2 | +1.000000 |
|---|--|---|--|--|----|----|----|----|--------------|-------------|--|--|--|---|----------|--------------------|---|---|---|---|----|----|--------------|-------------|---|----------|-------|---|---|---|---|---|---|-----|---|---|----------|--------|----|----|----|---|---|---|-------|-----------|---|----------|---------|----|----|----|---|---|---|-------|-----------|---|----------|---------|----|----|----|---|---|---|------|-----------|---|----------|---------|----|----|---|---|---|---|-----|-----------|---|----------|---------|----|---|---|----|---|---|-----|-----------|---|-----------|---------|----|----|---|----|---|---|-----|-----------|---|----------|---------|----|----|---|---|---|---|-------|-----------|---|----------|---------|----|----|----|---|---|---|-----|-----------|----|----------|---------|----|---|----|---|---|---|-----|-----------|----|-----------|---------|----|----|---|---|---|---|-------|-----------|----|----------|---------|----|---|----|---|---|---|-------|-----------|----|-----------|---------|---|----|---|----|---|---|-----|-----------|----|-----------|---------|----|----|---|----|---|---|-------|-----------|----|----------|---------|----|----|---|---|---|---|-----|-----------|----|----------|---------|----|----|----|---|---|---|-------|-----------|--|--|---------|----|----|----|---|---|---|-------|-----------|--|--|---------|---|---|----|----|---|---|-----|-----------|--|--|---------|----|----|----|---|---|---|-------|-----------|--|--|---------|----|----|----|---|---|---|-------|-----------|--|--|---------|----|---|----|----|---|---|------|-----------|--|--|---------|----|----|----|---|---|---|-------|-----------|--|--|---------|----|----|---|---|---|---|-----|-----------|--|--|---------|----|----|---|----|---|---|-------|-----------|--|--|---------|----|----|---|----|---|---|------|-----------|--|--|---------|----|---|----|---|---|---|-------|-----------|--|--|---------|----|----|---|---|---|---|--------|-----------|--|--|---------|----|---|----|---|---|---|-----|-----------|--|--|---------|---|----|----|---|---|---|-----|-----------|--|--|---------|----|----|---|---|---|---|-------|-----------|--|--|---------|----|----|---|----|---|---|------|-----------|--|--|---------|----|---|---|----|---|---|-----|-----------|--|--|---------|----|----|---|---|---|---|------|-----------|--|--|----------|---|----|----|---|---|---|-----|-----------|--|--|----------|----|----|----|---|---|---|------|-----------|--|--|----------|----|----|----|---|---|---|-------|-----------|--|--|----------|----|----|---|----|---|---|-------|-----------|--|--|----------|----|---|---|---|---|---|-----|-----------|
| Indigestibility $\xi(N)$ of the natural numbers 1-16 | Complete Intraoctave Intervals upwards of Harmonicity 0.05 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| N | $\xi(N)$ | Interval-size (Ct) | 2 | 3 | 5 | 7 | 11 | 13 | Number-ratio | Harmonicity | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0.000000 | 0.000 | 0 | 0 | 0 | 0 | 0 | 0 | 1:1 | ∞ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 1.000000 | 70.672 | -3 | -1 | +2 | 0 | 0 | 0 | 24:25 | +0.054152 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 2.666667 | 111.731 | +4 | -1 | -1 | 0 | 0 | 0 | 15:16 | -0.076531 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 2.000000 | 182.404 | +1 | -2 | +1 | 0 | 0 | 0 | 9:10 | +0.078534 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 6.400000 | 203.910 | -3 | +2 | 0 | 0 | 0 | 0 | 8:9 | -0.120000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 3.666667 | 231.174 | +3 | 0 | 0 | -1 | 0 | 0 | 7:8 | -0.075269 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | 10.285714 | 266.871 | -1 | -1 | 0 | +1 | 0 | 0 | 6:7 | +0.071672 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 8 | 3.000000 | 294.135 | +5 | -3 | 0 | 0 | 0 | 0 | 27:32 | -0.076923 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 9 | 5.333333 | 315.641 | +1 | +1 | -1 | 0 | 0 | 0 | 5:6 | -0.099338 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | 7.400000 | 386.314 | -2 | 0 | +1 | 0 | 0 | 0 | 4:5 | +0.119046 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 11 | 18.181818 | 407.820 | -6 | +4 | 0 | 0 | 0 | 0 | 64:81 | +0.060000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 | 4.666667 | 427.373 | +5 | 0 | -2 | 0 | 0 | 0 | 25:32 | -0.056180 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 13 | 22.153846 | 435.084 | 0 | +2 | 0 | -1 | 0 | 0 | 7:9 | -0.044024 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 14 | 11.285714 | 470.781 | -4 | +1 | 0 | +1 | 0 | 0 | 16:21 | +0.058989 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 15 | 9.066667 | 498.045 | +2 | -1 | 0 | 0 | 0 | 0 | 3:4 | -0.214286 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 16 | 4.000000 | 519.551 | -2 | +3 | -1 | 0 | 0 | 0 | 20:27 | -0.060976 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 568.717 | -1 | -2 | +2 | 0 | 0 | 0 | 18:25 | +0.052265 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 582.512 | 0 | 0 | -1 | +1 | 0 | 0 | 5:7 | -0.059922 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 590.224 | -5 | +2 | +1 | 0 | 0 | 0 | 32:45 | +0.059761 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 609.776 | +6 | -2 | -1 | 0 | 0 | 0 | 45:64 | -0.056391 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 617.488 | +1 | 0 | +1 | -1 | 0 | 0 | 7:10 | -0.056543 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 680.449 | +3 | -3 | +1 | 0 | 0 | 0 | 27:40 | +0.057421 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 701.955 | -1 | +1 | 0 | 0 | 0 | 0 | 2:3 | +0.272727 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 729.219 | +5 | -1 | 0 | -1 | 0 | 0 | 21:32 | -0.055703 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 764.916 | +1 | -2 | 0 | +1 | 0 | 0 | 9:14 | +0.060172 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 772.627 | -4 | 0 | +2 | 0 | 0 | 0 | 16:25 | +0.059524 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 792.180 | +7 | -4 | 0 | 0 | 0 | 0 | 81:128 | -0.056504 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 813.686 | +3 | 0 | -1 | 0 | 0 | 0 | 5:8 | -0.106385 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 884.359 | 0 | -1 | +1 | 0 | 0 | 0 | 3:5 | +0.110294 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 905.865 | -4 | +3 | 0 | 0 | 0 | 0 | 16:27 | +0.083333 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 933.129 | +2 | +1 | 0 | -1 | 0 | 0 | 7:12 | -0.066879 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 968.826 | -2 | 0 | 0 | +1 | 0 | 0 | 4:7 | +0.081395 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 996.090 | +4 | -2 | 0 | 0 | 0 | 0 | 9:16 | -0.107143 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 1017.596 | 0 | +2 | -1 | 0 | 0 | 0 | 5:9 | -0.085227 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 1086.269 | -3 | +1 | +1 | 0 | 0 | 0 | 8:15 | -0.082375 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 1125.338 | +4 | +1 | -2 | 0 | 0 | 0 | 25:48 | -0.051370 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 1137.039 | +1 | +3 | 0 | -1 | 0 | 0 | 14:27 | -0.051852 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 1200.000 | +1 | 0 | 0 | 0 | 0 | 0 | 1:2 | +1.000000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| $\mathcal{H}(P, Q) = \frac{\text{sgn}(\xi(Q) - \xi(P))}{\xi(P) + \xi(Q)}$ <p>whereby $\text{sgn}(x) = -1$ for $x < 0$, else $\text{sgn}(x) = +1$</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

In 1986 I installed these formulae in a computer program called "Autobusk" (finally completed in 2002), which I have used on numerous occasions to compose music in real-time.

Parallel to the development of these harmonic formulae, I came up with a set of formulae for what I call metric *indispensability*, the relative metric significance of a pulse within a metre. These values, unique to each pulse, are shown in Fig.10 along with a demonstration at bottom right of the values for the metres $3/4$ and $6/8$ on an eighth-note level: the indispensabilities for the former are 5 0 3 1 4 2, and for the latter 5 0 2 4 1 3, clearly reflecting the respective significance of the six pulses therein. The process of *rhythmic dilution* also shown at bottom right, removing attacks in order of increasing indispensability, further demonstrates the system, also part of Autobusk.

See also references [4] and [5].

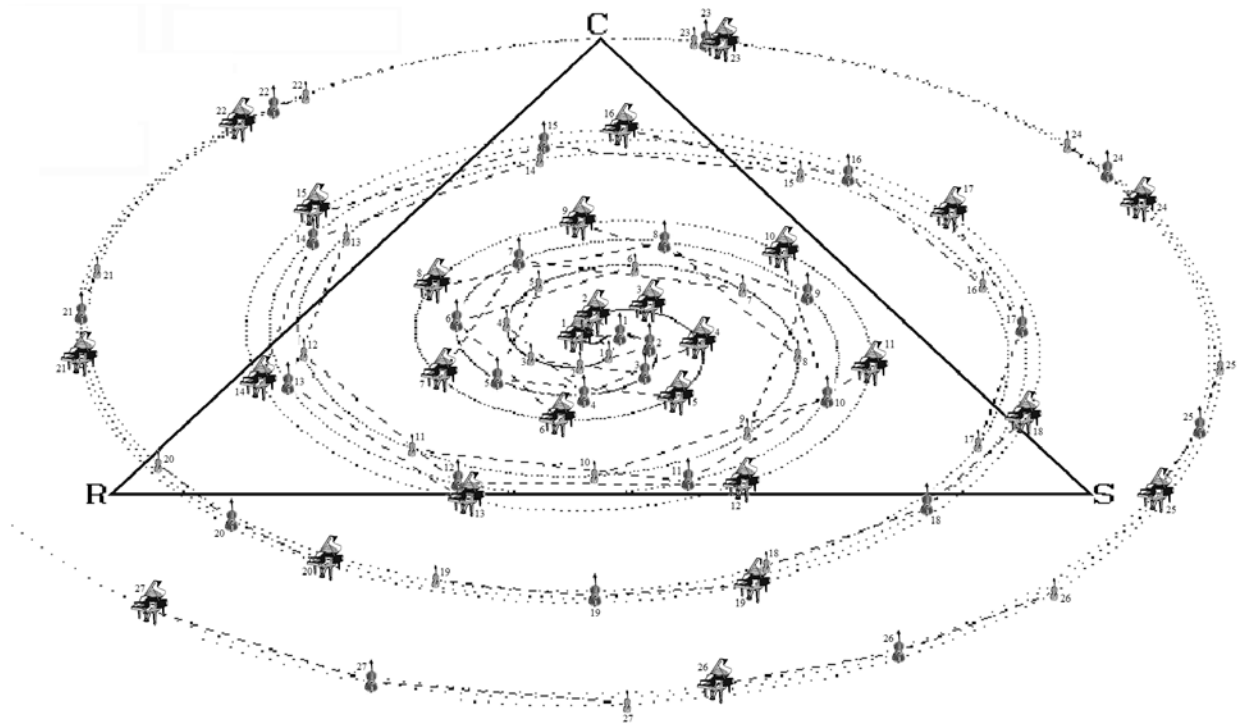
Fig.10 – Formulae for Metric Indispensability, with a table and an example of rhythmic dilution at bottom right.

| $\Psi_z(n) = \sum_{r=0}^{z-1} \left\{ \prod_{i=0}^{z-r-1} p_i \Psi_{p_{z-r}} \left(1 + \left[1 + \frac{(n-2) \bmod \prod_{j=1}^z p_j}{\prod_{k=0}^r p_{z+1-k}} \right] \bmod p_{z-r} \right) \right\}$ <p>whereby (all variables being whole numbers):</p> <ol style="list-style-type: none"> $p_0 = p_{z+1} = 1$ n is the position in the bar of the pulse in question, starting at 1 p_j is the stratification divisor on level j z is the number of levels in the stratification $\Psi_p(x)$ is the Indispensability of the xth pulse of a first-order bar with the prime stratification p $u \bmod v$ is the remainder of the division $(u+mv)/v$, by sufficiently large m never negative $[x]$ is the whole-number component of x | | | | | | | | | | | | | | | | | | | | | |
|--|---|-------------------------|--|--------|-------------|---------|------------------------------|----------|------------------------------|---|--|---|--|---|--|---|--|---|--|---|--|
| <p>If $p=2$, then $\Psi_p(n) = p-n$;</p> <p>otherwise if $n=p-1$, then $\Psi_p(n) = [p/4]$</p> <p>or else $\Psi_p(n) = [q+2\sqrt{q+1}/p]$</p> <p>whereby</p> <ol style="list-style-type: none"> n is the position in the bar of the pulse in question, starting at 1 $q = \Psi_{p-1}(n - [n/p])$ $\Psi_{p-1}(x)$ gives the Indispensabilities for a bar of pulses numbering $p-1$, factorised and stratified with primes in decreasing order of size <p>See also 5. and 7. in the previous diagramme</p> | <table border="1"> <thead> <tr> <th colspan="2">Rhythmic Dilution Chart</th> </tr> <tr> <th>metre:</th> <th>$3/4$ $6/8$</th> </tr> </thead> <tbody> <tr> <td>pulses:</td> <td>1 2 3 4 5 6 1 2 3 4 5 6</td> </tr> <tr> <td>indisp.:</td> <td>5 0 3 1 4 2 5 0 2 4 1 3</td> </tr> <tr> <td>6</td> <td></td> </tr> <tr> <td>5</td> <td></td> </tr> <tr> <td>4</td> <td></td> </tr> <tr> <td>3</td> <td></td> </tr> <tr> <td>2</td> <td></td> </tr> <tr> <td>1</td> <td></td> </tr> </tbody> </table> | Rhythmic Dilution Chart | | metre: | $3/4$ $6/8$ | pulses: | 1 2 3 4 5 6 1 2 3 4 5 6 | indisp.: | 5 0 3 1 4 2 5 0 2 4 1 3 | 6 | | 5 | | 4 | | 3 | | 2 | | 1 | |
| Rhythmic Dilution Chart | | | | | | | | | | | | | | | | | | | | | |
| metre: | $3/4$ $6/8$ | | | | | | | | | | | | | | | | | | | | |
| pulses: | 1 2 3 4 5 6 1 2 3 4 5 6 | | | | | | | | | | | | | | | | | | | | |
| indisp.: | 5 0 3 1 4 2 5 0 2 4 1 3 | | | | | | | | | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | | | | | | | | |
| 1 | | | | | | | | | | | | | | | | | | | | | |

Musical Sources

The final example in this paper shows my use of existing pieces of music in a sort of microcollage, as in the piano trio *1981* (1981): piano trios by the three composers Muzio Clementi (*La Chasse* in C, 1788), Robert Schumann (*Trio No.2* in F, 1847) and Maurice Ravel (*Trio* in A minor, 1914), respectively, one movement each, formed the point of departure for this composition. The compositional process is as follows. Each of the three instruments, violin, cello and piano, describes an individual spiral starting at the centre of an equilateral triangle with the composers Clementi, Schumann and Ravel at the apices marked "C", "S" and "R", as shown in Fig.11. The numbers alongside the images of the three instruments are score markers, multiples of 18 bars. The distance of the instruments' position from an apex at any given time is related to the amount of the corresponding composer's music in the general mix. Thus, at the beginning of the piece, at the triangle centre, 33% of the notes in the score come from each of the three composers. However at bar 410, just to the left of marker 23, all three instruments are together at the Clementi apex, meaning that the score contains 100% of his music at this point, together with a small percentage of the music of the other two composers, more than zero since they are not infinitely distant.

Fig.11 – Triangle and Spiral used for the composition of *1981*



This highly compressed account of my use of various sources for the composition of music shows how music can originate outside of the conventional means of mental imagination of sound prior to its committal to paper or disk. Yet, I have never allowed the results of the processes outlined above to be unmusical as far as my own taste is concerned; the results were always subjected to acute aesthetic examination and permitted their existence and publication only when I was musically satisfied.

References

- [1] C. Barlow, *On the Spectral Analysis of Speech for Subsequent Resynthesis by Acoustic Instruments*, Mnemosyne Volume II, Bourges, France, 1996
- [2] C. Barlow, *Songs Within Words*, in *Samuel Beckett and Music*, ed. Mary Bryden, Oxford, UK, 1998.
- [3] C. Barlow, *Auf der Suche nach dem reell Globalen - Zur physischen Ertastung der Erde in Raum und Zeit* in *Lab*, Year book 2000 for Art and Machines, Media and Arts Academy Cologne, Walther König Verlag, Cologne, Germany, 2000.
- [4] C. Barlow, *AUTOBUSK Manual*, Report No.44 of the Musicological Institute/ Musikinformatik & Medientechnik of Gutenberg University, Mainz, Germany, 2001
- [5] C. Barlow, *On the Quantification of Harmony and Metre* in *THE RATIO BOOK*, ed. C. Barlow, Feedback Papers No.43, Feedback Studio, Cologne, Germany, 2001
- [6] C. Barlow, *ISIS, an Alternative Approach to Sound Waves*, Proceedings of the International Computer Music Conference, Barcelona, Spain, 2005
- [7] C. Barlow, *Musica Visualis: On the Sonification of the Visual and the Visualisation of Sound*, Proceedings of the Symposium on Systems Research in the Arts and Humanities, Baden-Baden, Germany, 2008
- [8] C. Barlow, *On Music Derived from Language*, Proceedings of the 3rd International Symposium on Systems Research in the Arts and Humanities, Baden-Baden, Germany, 2009
- [9] C. Barlow, *Mathematics as the Source of Music Composition*, Proceedings of the 1st International Symposium on Music/Sonic Art, Baden-Baden, Germany, 2010
- [10] C. Barlow, *Making Music out of Field Recordings*, Proceedings of the 2nd International Symposium on Music/Sonic Art, Baden-Baden, Germany, 2011