On Ramifications of Intonation

In the context of music, intonation is a subject which has interested me greatly since August 1967, when I visited Kabul and heard Afghan popular music, rife with its quarter tones, for the very first time. I had been composing music for some ten years then and found this tuning fascinating. About a year earlier I had been introduced to North Indian classical music and the study of it, which, its twelve scale degrees cognate with the twelve notes I had known in European classical music since I was a small child, did not seem to me to be particularly striking in its tuning. It was my second visit to Kabul in 1975, followed by a journey through Afghanistan, Iran and Turkey, that triggered in me a serious desire to employ quarter tones in composition. I did this with my piano piece *Çoğluotobüsişletmesi* (1978).

This was not the first time I departed from the principle of the equal-tempered twelve-tone chromatic scale; in 1972 I started a series of (to date) nine pieces named *...until...* that were conceptually based on just intonation with scale-degree frequencies in integer relationships n:n±1 to a central drone. Yet the first six of these pieces stopped short of the ratio 6:7 and all but only one were written for tempered instruments – more about this apparent discrepancy later; it was in Version 7 for guitar of 1981 that I made so bold as to expressly incorporate the interval 6:7 and implement just intonation.

Let me touch here on the generally accepted meaning of "intonation" in music. Among the definitions accorded to the term by several sources, a particular one stands out: that of accuracy or "in-tuneness" of pitch rendition. Here are some excerpts from the internet, retrieved on 23 December 2013, in order of increasing brevity:

http://www.audioenglish.org/dictionary/intonation.htm: the production of musical tones (by voice or instrument); especially the exactitude of the pitch relations.

http://en.wikipedia.org/wiki/Intonation_(music): Intonation, in music, is a musician's realization of pitch accuracy, or the pitch accuracy of a musical instrument.

http://www.oxforddictionaries.com/definition/english/intonation: accuracy of pitch in playing or singing, or on a stringed instrument such as a guitar.

http://de.wikipedia.org/wiki/Intonation_(Musik): Mit Intonation wird in der Praxis des Musizierens die Feinabstimmung von Tonhöhen bezeichnet.

http://www.thefreedictionary.com/intonation: A manner of producing or uttering tones, especially with regard to accuracy of pitch.

http://de.thefreedictionary.com/Intonation: die Genauigkeit in der Tonhöhe beim Instrumentalspiel und Gesang.

http://www.merriam-webster.com/dictionary/intonation: the ability to play or sing notes in tune.

This approach shifts the focus to the terms "pitch" and "in tune", of which "pitch" is unproblematic, being associated in a musical context with the physical frequency of a sound, measurable in Hertz. The meaning of "in tune", however, could be a matter of debate. In my understanding, it suffices to consider "intonation" simply as the judicious placement of frequency. This wide generality opens up numerous fields of application.

In the following, I will attempt to exemplify intonation in terms of **Pitch** in a broad sense, as well as of **Harmony** and **Timbre**. It will furthermore be shown – without my going into detail – that the realms of **Melody**, **Rhythm**, **Sonority** and **Speech** are contingent on these ruminations, as illustrated in Figure 1 below.

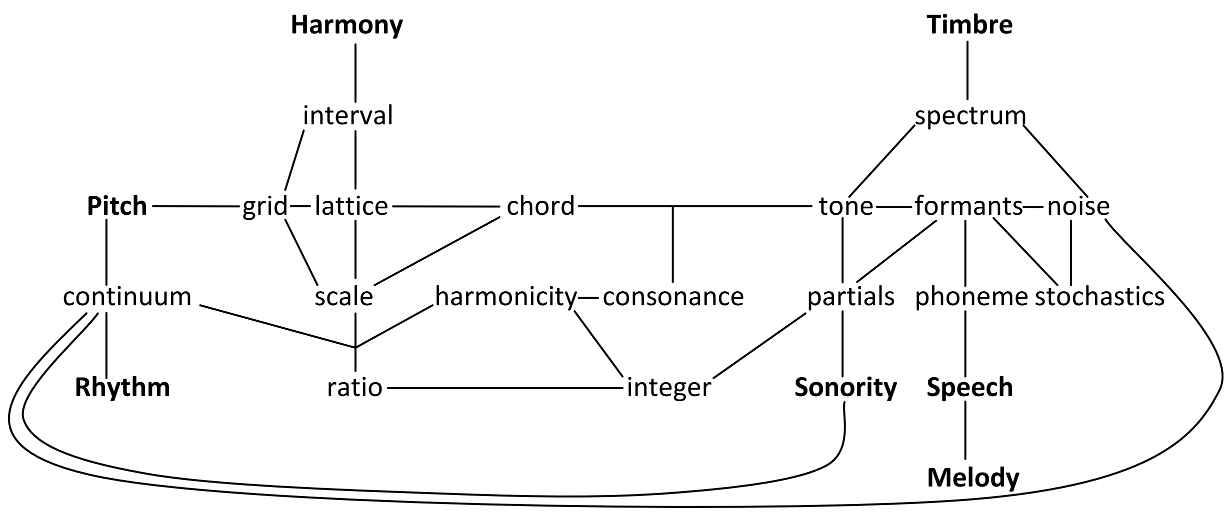

Fig. 1 – Network of numerous entities relevant to intonation

1. Pitch

Ever since the introduction of electronic music in the early 20th century it has been possible to make music in pitch space without the concept of sounds being "in" or "out of tune". For my electronic piece *Sinophony I* (1970) for instance, I divided the space between 36 Hz and 14400 Hz into 79 equal steps of 1.313 semitones without any regard to this pitch distance and multiples of it as musical intervals. In *Sinophony II* (1972) I chose a grid of perfect fifths and attendant pitches, in theory stretching infinitely but practically, given the equipment I had, from 17 Hz to 17kHz, without consciously using the fifth as a harmonic or timbral interval. In both cases had I set up an arbitrary grid in the neutral continuum of pitch space. Atonal dodecaphonic systems of the early 20th century arguably used or attempted to use the equal-tempered twelve-tone chromatic scale as a similarly non-harmonic pitch grid.

In some works of mine, non-grid usage of this continuum was effected by glissandi, occasionally reaching down far below 20 Hz into the domain of rhythm. This pitch-rhythm contiguity has been well established and demonstrated by Stockhausen[1] and others. A technique I developed in 2001, called Intra-Samplar Interpolating Sinusoids (ISIS)[2], can algebraically convert any digitised sound wave into an ultrasonic sine-tone sequence and vice versa, establishing a melody-timbre contiguity within the pitch continuum. More about ISIS later.

2. Harmony

In the pitch continuum, one can set up a lattice of harmonically defined intervals which can give rise to harmonically relevant scales and/or chords. Note that the adjacent labels "grid" and "lattice" near the left of Figure 1, though related, differ in implication: harmonic irrelevance by the former, harmonic relevance by the latter.

What is harmonic relevance? Authorities from Pythagoras to Partch and beyond have consistently maintained that intervals formed from frequency ratios containing small numbers (e.g. perfect octave 1:2 and fifth 2:3) are more "harmonic" than those derived from larger ones (e.g. minor second 15:16 and augmented fourth 32:45). The word "consonant" is frequently used for this property, but for reasons to be explained later, I will adhere in this context to the terms "harmonic" and "harmonicity". Several writers of the last century and a half have listed twelve chromatic intervals, sometime by name, sometimes as ratios, in decreasing order of harmonicity, starting with the octave and ending with the augmented fourth. But is the 3-limit Pythagorean major third 64:81 as harmonic as its 5-limit overtone series counterpart 4:5? (the prime limits refer to the highest prime number contained in the ratio components).

In 1975, during my trip through the Middle East, I wondered where quarter tones would fit into this scheme of things. Not finding any in-depth literature on the subject, I resolved to delve into the mathematics and find a solution myself. Certainly, the great Leonhard Euler had developed his gradus suavitatis and his totient function almost three centuries before, but they seemed too undifferentiated for my purposes. The result was my 1978 algebraic function of *numerical indigestibility*, ξ(N), based on the size and the divisibility of an integer (examples: ξ(5) = 6.4, ξ(6) = 3.7, ξ(7) = 10.3, ξ(8) = 3.0) on which I built a function H(P,Q) of *interval harmonicity* (examples: H(27,32) = –0.08, H(5,6) = –0.1, H(9,11) = +0.04, H(4,5) = +0.12, H(64,81) = +0.06). The quarter-tone neutral third 9:11 is listed here, as are the (unequally harmonic) 3- and 5-limit major thirds mentioned above. With computer help and using a probabilistic random-number system based on this harmonicity function I created pitch fields varying smoothly in tonal strength from 0 (atonal) to 1 (fully tonal). The resulting piano piece, *Çoğluotobüsişletmesi*, requires the strings for the notes B, D, E and F-sharp in all octaves to be tuned down a quarter tone.

Subsequently, I used the harmonicity function to rationalise the tuning of scales known only by interval size. The computer program I wrote for the purpose, given an input of e.g. 0, 1, 2, 3, ... 11, 12 (the equal-tempered twelve-tone chromatic scale in semitones), searches the pitch continuum in the immediate vicinity of the input and – given certain constraints – here outputs 1:1, 8:9, 4:5, ... 8:15, 1:2 (the classical just-intoned rendition of the same scale). Any scale or chord, even "microtonal" ones with intervals outside the grid of the equal-tempered twelve-tone scale, can be similarly rationalised. In 2001, using the calculated harmonicities of the output ratios I made a two-dimensional model of the tuning using so-called *multidimensional scaling* (MDS), in which the proximity of the ratios is proportional to their mutual harmonicity. Figure 2 shows a two-dimensional map of a thus rationalised equal-tempered twelve-tone chromatic scale.

15:16

5:8 3:4

5:6 1:2 3:5

5:9 2:3 4:5

8:9 8:15

32:45

Fig. 2 – Two-dimensional multidimensionally scaled model of the rationalised equal-tempered twelve-tone scale

In 2012 I went a step further with this method, developing a system called *Chorale Synthesis*. By studying the movement of chords shown as polygons on a MDS map of the rationalised (i.e. just-intoned) pitch material of a Bach chorale, I was able to postulate two rules to automatically generate chorales based on a network of interval ratios: 1) in the MDS map of the interval repertoire, find chords with harmonicities proportional to the respective pulse strength, 2) each chord must have at least one note in common with the preceding and the following chord, if there be one. The chorale-synthesised organ piece *Für Simon Jonasohn-Stein* is based on 79 scale degrees spanning 4½ octaves (55 semitones) generated by primes raised to powers within the limits $2^{\pm 9}$, $3^{\pm 6}$, $5^{\pm 2}$ and $7^{\pm 1}$. The music, while not driven by functional harmony in the classical sense, sounds indeed vaguely familiar. This was my first attempt to develop a functional harmonic grammar extending the prime limit beyond the classical 2, 3 and 5 to the next prime 7.

Though 79 scale degrees in a range of 55 semitones are counted here, the organ was not retuned but remained in its regular equal-tempered twelve-tone tuning. Why this apparent discrepancy? I quote from my book *On Musiquantics* (1984-2012)[3]: "For purposes of further explanation, I draw upon three linguistic concepts: Phonetics, which describes the actual sounds, Semantics, which attempts (if need be through adjusted hearing) to make sense of the sounds, and Grammar, which through a set of rules can serve both the understanding of the sounds heard as well as their production for the purpose of comprehension. A pitch system, the grammar of which is effectively employed, can be grasped semantically, even if slight 'phonetic' deviations from the expected norm are present (similar to accent in language). An example: Bach also makes sense on instruments with modern tuning. In short: Grammar assists Semantics in comprehending Phonetics." Some of my pieces can be played at will with equal-tempered 12-, 17- or 24-tone intonation. And it has been shown that the intonation of North Indian classical modes vary slightly from school to school[4] and even among performers in any one school.

3. Timbre

The harmonicity function described above does not depend in any way on the timbre of the instrument used. Indeed, the harmonies of a Bach chorale would be fully recognizable regardless (within limits) of the instrumentation. But timbre is indeed a very important parameter. Legendary work by researchers such as Plomp[5], Grey[6], Wessel[7] and others in the 1970s involved the multidimensional scaling of timbral similarity as perceived by invited listeners; their work led to the identification of parameters variously called *attack quality* or *rise time*, *brightness* or *spectral centroid*, *spectral flux*. One can visualise musical instruments as points in a three-dimensional space bounded by these parameters. Accompanied by other characteristics, they have been included as audio descriptors in the list of MPEG-7 audio standards, first released in 2002. I look forward to working with this fascinating approach to timbre some day.

A useful representation of timbre is the *spectrum*, a Cartesian depiction of a sound's loudness against frequency. The are two basic types of spectrum which can exist singly or in combination (refer again to Figure 1): *tone spectra*, which comprise a sounding together of independent sine tones and *noise spectra*, in which the loudness curve is continuous. Tone spectra are *harmonic* when the frequency components are all exact integer multiples of a fundamental which may be present or absent (as in the case of the lowest tones of a piano). When the component frequencies, also called *partials*, are not multiples of a single fundamental, the tone spectrum is said to be *inharmonic*, as in the case of bells.

In 1965, the Dutch researchers Plomp and Levelt published a paper[8], a landmark in psychoacoustics, on what they called "Tonal Consonance" and has otherwise been called "Sensory Consonance" or simply "*consonance*". This term has been used for centuries for the concept of harmonicity, and it was only in the mid-20th century that the two have been shown to be distinct, but related in central audible pitch space. The adjacent entities "harmonicity" and "consonance" can be seen in the middle of Figure 1.

In opposition to harmonicity, consonance (its converse is dissonance) was shown by Plomp and Levelt to be intrinsically dependent on timbre; when two partials of one or more tone spectra are at a certain critical distance from each other (e.g. the fifth partial B of the cello G-string and the eighth partial C of the cello C-string), they cause a type of friction or dissonance which can vary from a maximum at the critical distance to none at all if the partials form a unison or near-unison or are further away from each other. This friction occurs in the inner ear on the basilar membrane, the structure of which causes the critical distance for maximum dissonance to vary from about two-thirds of a semitone at 2 kHz to as much as a major third at 100 Hz; this distance, according to Plomp and Levelt, is one-quarter of the so-called *critical band*[9], the unit of the *Bark Scale*[10] of subjective pitch. Plomp and Levelt's method, adding dissonances at all points of friction, can help to evaluate the dissonance of any chord, given its spectral definition; I availed of it in 1978 while composing *Çoğluotobüsişletmesi* and more recently wrote a computer program called *Dissonometer* (2003) which employs an augmentation of the method.

Both tone and noise spectra can exhibit *formants*, frequency zones of increased loudness. This phenomenon is particularly important for the recognition of *phonemes*, e.g. the sound [a] in "father", in which important formants are found at the frequencies 800 Hz and 1200 Hz, fairly independently of the age and sex of the speaker, or the sound [i] as the vowel in "feet", with formants at 300 Hz and 3000 Hz. The intonation of phonemes is effected by the tongue and the vocal and nasal tracts. The phonetic domain forms a transition to that of human speech in general, where the term "intonation" is also used, but pertaining to alterations of the frequency of the spectral fundamental, where the vocal cords are the effective organs. And long ago, speech intonation may have been one of the origins of melody.

An interesting aspect of chord and timbre is their mutually distinct identity. If a chord consists only of tones forming a harmonic series, as do the partials forming a harmonic timbre, could it be perceived as a single, albeit complex, tone? The answer is: it could, particulary if its components are sine-tone-like. In 1982 I set out to write an ensemble piece, *Im Januar am Nil*, in which a set of natural harmonics played on separate string instruments would blend to form recognisable spectra derived from analyses of words spoken in German, containing no plosives, fricatives or other noise components. An example is "Um null Uhr am nahen Mahnmal meine Meinung murmeln", a sentence consisting only of vowels, nasals and a lateral. The fundamental frequency in this piece ranged from 55 Hz (A1) to 98 Hz (G2) with the highest partial number at 60, around 3.3 kHz, played as the 5th natural harmonic on the violin E-string. I call this technique *Synthrumentation*, from "synthesis through instrumentation".

This timbral aspect of composition obviates the relevance of the partial numbers' divisibility: the mutual difference in the effect of partials 58, 59 or 60 as contributors to formants is neglible, and thus the exact number is unimportant. This leads to the question of why some composers consciously use high partial numbers like "57" or "49". In my opinion, high primes are harmonically meaningless; it is difficult enough to base harmony on the prime number 7 convincingly, let alone 11 or 13, and in my compositions *...until... #7* and *Für Simon Jonasohn-Stein* I trod on this ground no less than gingerly. I would not try 11 before I feel I have mastered 7 (which is not yet the case). Once while discussing this issue with Marc Sabat it struck me that the conscious and explicit use of high primes was due to something quite removed from harmony and harmonicity, and belonging more to the realm of *sonority*. It would seem to me that the music aspired to in this case would appeal less through functional harmony, chords and modulation than through the sound itself. Indeed, I now feel that in my piece *Çoğluotobüsişletmesi* of 1978 I approached quarter tones more from the angle of sonority than that of harmony, even though the implementation of the latter – including the prime number 11 for the only time in my life – helped me to compose the piece.

Returning again to noise spectra: it was in the late 1960s when making my first electronic pieces that I began to doubt the adage that white noise was the sum of all frequencies. Surely "all frequencies" would be infinite in number, I felt, forcing the loudness of each of them down to zero! Put another way, an infinite number of frequencies, however infinitesimal each one's non-zero loudness is, would in sum total have to be infinitely loud. That a segment of white noise contains a non-infinite frequency selection is demonstrated for instance by how obvious the loops sound if the segment is continually repeated.

An infinite number of simultaneous frequencies is physically impossible. I opted for another point of view: white noise as a single frequency moving as randomly as possible to all imaginable values at an intangibly high speed. Indeed, from this angle, any noise is the result of a probabilistic *stochastic* process, the spectrum representing the probability of any given frequency being touched on at any given time.

Experiments with reverbed sine glissandi in the 70s, with dense MIDI tone clouds in the 80s, led in 2001 to the method described earlier: *ISIS*, for *Intra-Samplar Interpolating Sinusoids*[2]. Given a stream of samples with values from -1 to +1, the algebraic ISIS formula calculates the frequency of imaginary sine tone segments stretching across empty time space from one sample to the next. The resulting – always microtonal – melody, centred around the sampling rate in ultrasonic pitch space, can be transposed down, slowed and stretched to form an audible sine-tone sequence. Transformed back by the reverse formula, the original sound can be regained from the ISIS melody. As shown to the right and left in Figure 1, the entities "formants" and "noise" described in this section are instrinsic to the harmonically neutral pitch-space continuum first described above in section 1. "Sonority" also seems to me to have links to the pitch continuum.

Thus intonation can be addressed according to considerations of pitch space, subjective pitch, harmony, consonance, sonority, phonetics, speech, melody and possibly further entities in the microtemporal domain of frequency. And pulse as infrasonic frequency in the pitch-rhythm continuum could indicate intonation in the macrotemporal domain of time in the form of tempo and rhythm.

References

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